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The Theory of Time[1]

What we would like to do is formulate a theory in which the time cost of goods is part of the choice process of consumers. We want to know how the ordinary demand curve that we estimate is affected by the inclusion of time as a constraint.

This is a review of the theory of time used in consumption as developed by Becker. Consider the simple maximization problem in two goods:

$$\max U = U(x_1, x_2) \;\; \text{subj.to} \;\; wT + E = \sum_{i=1,2} (P_i + wt_i)x_i$$

where w is the wage rate, T is total time, E is non-wage income, P_i are money prices, and t_i are time prices. The budget constraint can be broken into two parts, the money budget $\Sigma P_i x_i$, and the time budget. The time budget has several components that Becker has melded nicely. There is the time spent consuming, $\Sigma t_i x_i$; the value of this time is $w\Sigma t_i x_i$. The implied labor-leisure tradeoff comes from the fact that $w(T- \Sigma t_i x_i) + E = \Sigma P_i x_i$. Leisure is not a good in the model, nor is work a bad. Indeed, the time spent consuming is not a good or a bad. It is a constraint.

The FOC are:

$$\begin{aligned} &U_1 - \lambda(P_1 + wt_1) = 0 \\ &U_2 - \lambda(P_2 + wt_2) = 0 \\ &wT + E - \sum_{i=1,2} (P_i + wt_i)x_i = 0 \end{aligned}$$

Let F_i stand for the full price of good i, that is, the regular money price plus the wage-valued opportunity cost of time price. The SSOC are then

$$H = \begin{vmatrix} U_{11} & U_{12} & -F_1 \\ U_{21} & U_{22} & -F_2 \\ -F_1 & -F_2 & 0 \end{vmatrix}$$

The FOC and SSOC combine to generate demand curves of the sort

$$x_i^* = x_i^*(P_1, P_2, t_1, t_2, w, E)$$

These are observable and well defined, as we will see when we look at the comparative statics.

The comparative statics of the problem are shown by the following matrix:

[1] Becker, G.S. "A Theory of Allocation of Time" in Economic Journal. Vol 75, 1965. p493-517. Silberberg (1990) (2001) section 11.4; Nicholson (1998) p175-177; Nicholson (2000) p438-444.

$$\begin{bmatrix} U_{11} & U_{12} & -F_1 \\ U_{21} & U_{22} & -F_2 \\ -F_1 & -F_2 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial x_1^*}{\partial P_1} & \frac{\partial x_1^*}{\partial t_1} & \frac{\partial x_1^*}{\partial w} & \frac{\partial x_1^*}{\partial E} \\ \frac{\partial x_2^*}{\partial P_1} & \frac{\partial x_2^*}{\partial t_1} & \frac{\partial x_2^*}{\partial w} & \frac{\partial x_2^*}{\partial E} \\ \frac{\partial \lambda^*}{\partial P_1} & \frac{\partial \lambda^*}{\partial t_1} & \frac{\partial \lambda^*}{\partial w} & \frac{\partial \lambda^*}{\partial E} \end{bmatrix} = \begin{bmatrix} \lambda & \lambda w & \lambda t_1 & 0 \\ 0 & 0 & \lambda t_2 & 0 \\ x_1^* & x_1^* w & \sum t_i x_i - T & -1 \end{bmatrix}$$

Solving by Cramer's rule gives the following:

$$\frac{\partial x_1^*}{\partial E} = -\frac{H_{31}}{H} \qquad 1$$

While this term cannot be signed, it is the normal income effect. Since the model breaks income into two parts, wage income and other, non-labor-time income, and since the wage rate is intermingled with income and consumption, the non-wage income term is the more straightforward expression. For normal goods, equation (1) is positive. A negative sign here would imply an inferior good. The demand curves allow for a direct estimate of this value when they are specified as per this model.

The principal question in any consumption model is the slope of the demand curve. This is given by:

$$\frac{\partial x_1^*}{\partial P_1} = \lambda \frac{H_{11}}{H} + x_1^* \frac{H_{31}}{H} = \lambda \frac{H_{11}}{H} - x_1^* \frac{\partial x_1^*}{\partial E} \qquad 2$$

which can be rewritten in our usual way:

$$\frac{\partial x_1^*}{\partial P_1} + x_1^* \frac{\partial x_1^*}{\partial E} = \lambda \frac{H_{11}}{H} \qquad 3$$

This is the normal demand curve expression. Equation (3) is the standard Slutsky equation. The ordinary price effect plus the weighted income effect must be negative. The right-hand side of (3) is negative. In the general case, this is true because the principal minors of the SSOC Hessian determinant alternate in sign. In the two good case, H_{11} is composed solely of parameters and can be seen to be negative by inspection.[2]

Next, let's look at the slope of the demand curve in terms of the time cost of a good. This is given by:

$$\frac{\partial x_1^*}{\partial t_1} = w\lambda \frac{H_{11}}{H} + w x_1^* \frac{H_{31}}{H} = w\left[\lambda \frac{H_{11}}{H} - x_1^* \frac{\partial x_1^*}{\partial E}\right] = w \frac{\partial x_1^*}{\partial P_1} \qquad 4$$

[2] The model is standard in other ways, too. The demand curves are homogeneous of degree zero in all money components. This means that the sum of the elasticities must be zero.

This is the headline result of the model. The slope of the demand curve in terms of time is simply the slope of the demand in terms of money prices times the wage rate. If the cost of consuming a good increases by one hour, then the effect on consumption will be the same as if the money price went up by the hourly wage rate. The implication of this result is that we can draw the demand curve in the normal fashion and reference price by the money price or the full price (money price plus the time price times *w*). If the full price is observed, then an increase in the money price or an increase in the time price moves us along the demand curve. If only the money price is observed, then a change in the time price will shift the demand curve. Even so, if the time price is not observed, the money price demand curve is the reference point for pricing (by firms or taxing by government) decisions.

Lastly, it is interesting to determine what happens when the wage rate changes. This is shown by:

$$\frac{\partial x_1^*}{\partial w} = \lambda t_1 \frac{H_{11}}{H} + \lambda t_2 \frac{H_{21}}{H} + \left(T - \sum t_i x_i\right)\frac{\partial x_1^*}{\partial E} \qquad 5$$

The first term on the right-hand side of (5) is negative, the second is positive (in the two-good world), and the third is conditioned by the income effect. The first term is the substitution effect. When the wage rate goes up, the opportunity cost of consuming the good is higher. The second term is the cross effect; the cost of consuming the other good is higher also. The best way to understand (5) is to think about the budget constraint. As the wage rate increases, the budget constraint shifts out. However, it does not shift out parallel. One or the other of the goods may be relatively time intensive (or costly). The budget shifts out favoring the less time intensive good. This effect is measured in the first two terms on the right-hand side of (5).

Next consider what happens to the slope of the ordinary demand curve when the wage rate increases. By Young's theorem, we can differentiate (5) to answer this question. This gives:

$$\frac{\partial^2 x_1^*}{\partial w \partial P_1} = \frac{\partial \lambda^*}{\partial P_1}\left[t_1 \frac{H_{11}}{H} + t_2 \frac{H_{21}}{H}\right] \qquad 6$$

Equation (6) is highly simplified and not necessarily correctly so. It assumes that all of the derivatives involving H^2 in the denominator can be considered trivially small. Also the sign of (6) depends on the partial of λ^* w.r.t. P_1 which cannot necessarily be determined. However, if we assume its sign is negative, which I think most likely,[3] then the sign of (6) depends on the bracketed term. The term in brackets has the same sign as the change in the slope of the budget constraint in $\{x_2, x_1\}$ space. When w changes, the budget constraint shifts out and pivots. The bracketed expression captures the pivot. If x_1 becomes relatively more costly because of the wage increase, then the budget constraint becomes more negatively sloped. This also causes the bracketed expression to be negative. The implication is that the change in the slope of the demand curve is positive; the demand curve becomes steeper (in $\{P,Q\}$ space) as the wage rate increases for goods that are relatively time intensive.

[3] As a price goes up, the marginal utility of money goes down because money is worth less.

Application of the model to a specific utility function allows us to verify our intuition to some extent. Consider the simple Cobb-Douglas function, x_1x_2.

$$\max_{\{x_1,x_2\}} U = x_1x_2 - \lambda(E + wT - \sum(P_i + wt_i)x_i)$$

The FOC are:

$$U_1 = x_2 - \lambda F_1 = 0$$
$$U_2 = x_1 - \lambda F_2 = 0$$
$$E + wT - \sum(P_i + wt_i)x_i = 0$$

Solving simultaneously gives:

$$x_1^* = \frac{E + wT}{2(P_1 + wt_1)}$$

The partial of this demand function with respect to the wage rate is:

$$\frac{\partial x_1^*}{\partial w} = \frac{T}{2(P_1 + wt_1)} - \frac{2t_1(E + wT)}{2(P_1 + wt_1)^2}$$

This shows the shift that we discussed above. When the wage rate goes up, demand shifts out because income is higher, but there is a mitigating effect because the time cost of consuming the good has gone up.

The partial with respect to price is:

$$\frac{\partial x_1^*}{\partial P_1} = -\frac{(E + wT)}{2(P_1 + wt_1)^2} = -\frac{x}{(P_1 + wt_1)}$$

Rewriting in elasticity terms:

$$\frac{\partial x_1^*}{\partial P_1}\frac{P_1}{x_1} = -\frac{P_1}{(P_1 + wt_1)}$$

This says that the money price elasticity is equal to the money price divided by the full price of the good. If the time price is zero, then the elasticity is equal to -1.

Differentiating elasticity with respect to the wage rate gives:

$$\frac{\partial\left[\frac{\partial x_1^*}{\partial P_1}\frac{P_1}{x_1}\right]}{\partial w} = \frac{t_1P_1}{(P_1 + wt_1)^2}$$

This says that as the wage rate goes up, demand becomes less elastic and this effect is related to the time price of the good.