INPUT DEMAND CURVES IN COMPETITIVE EQUILIBRIUM

The theory of competitive equilibrium has been examined at length in regard to the refutable implications it produces. Of continuing interest is the case of input demand curves.[1] Many different input demand curves can be defined. The simplest is that derived from the model of cost minimization subject to an output constraint. However, this formulation has limited empirical value because the firm's demand for inputs is affected by the output adjustments forced on the firm by competition. It has been shown many times that the input demand curves for the firm as it is moved from one competitive equilibrium to another are not necessarily downward sloping. The industry input demand, on the other hand, is always downward sloping under the assumption of identical firms.

This paper presents a new derivation of industry input demand that is notable because of its parsimony. The number of firms necessary to achieve the competitive equilibrium is modeled explicitly and differentiated with respect to the input price. The industry results then follow directly from the standard analysis for the representative firm.[2]

1. Firm-Level Input Demand Curves When Output is Constant and When Output Adjusts to the Competitive Equilibrium

The standard analysis starts with the problem of cost minimization subject to an output constraint.

$$\min C = \sum_{i=1}^{m} w_i x_i \quad \text{subj. to:} \quad q = f(x_1, \ldots, x_m)$$

Assuming that the sufficient second order conditions are satisfied, the first order conditions of this problem can be solved to yield input demand functions which have as arguments the input prices and the output level, *q*, i.e.,

$$x_i^* = x_i^*(w_1, \ldots, w_m, q) \qquad 1$$

These are the basic, firm-level demand curves. It is easily demonstrated that they are downward sloping, i.e., $\frac{\partial x_i^*}{\partial w_i} < 0$, for all *i*=1,*m*.

Next the theory examines the effect of an input price change on equilibrium output of competitive firm. This is accomplished by assuming that the firm operates at the point where marginal and average costs are equal. Using the cost function implied by the optimal input demand functions described by equation (1) we can write

$$MC(w_1, \ldots, w_m, q^*) = AC(w_1, \ldots, w_m, q^*) \qquad 2$$

where q^* is the output level that solves this equation. In this sense, q^* is a function of the remaining parameters in the problem, the input prices, i.e., $q^*(w_1,\ldots,w_m)$.

[1] See Ferguson and Saving, Bassett and Borcherding, Silberberg, and Heiner.

[2] The standard analysis is that first presented by Bassett and Borcherding, which is based on the envelope theorem generalized by Silberberg.

This formulation is used to determine how the long-run competitive equilibrium output level of the firm changes as an input price changes. Differentiating with respect to w_i gives,

$$\frac{\partial q^*}{\partial w_i} = \frac{\frac{\partial AC}{\partial w_i} - \frac{\partial MC}{\partial w_i}}{\frac{\partial MC}{\partial q}} \qquad 3$$

Substitute for $\partial AC / \partial w_i$ and $\partial MC / \partial w_i$ using the envelope theorem.

$$\frac{\partial q^*}{\partial w_i} = \frac{\frac{x_i^*}{q} - \frac{\partial x_i^*}{\partial q}}{\frac{\partial MC}{\partial q}} = \frac{\frac{x_i^*}{q}\left[1 - \varepsilon_{iq}\right]}{\frac{\partial MC}{\partial q}} \qquad 4$$

This is a familiar result which says that the effect on the competitive equilibrium output level of the firm that is associated with an input price change depends on the output elasticity of the input. If the input is superior, then the output level of the firm will fall as the input price increases. Otherwise the equilibrium output level will rise.

The fact that the equilibrium output level of the firm may increase opens the possibility that the input demand curve, based on q^* as opposed to an arbitrary output level, may not be downward sloping. To analyze this possibility substitute $q^*(.)$ into $x_i^*(.)$ which yields

$$x_i^{**} = x_i^*(w_1, \ldots, w_m, q^*).$$

Differentiating with respect to w_i gives the firm-level input demand curves when firms produce at the competitive equilibrium output level:

$$\frac{\partial x_i^{**}}{\partial w_i} = \frac{\partial x_i^*}{\partial w_i} + \frac{\partial x_i^*}{\partial q}\frac{\partial q^*}{\partial w_i} = \frac{\partial x_i^*}{\partial w_i} + \frac{\partial x_i^*}{\partial q}\left[\frac{\frac{x_i^*}{q} - \frac{\partial x_i^*}{\partial q}}{\frac{\partial MC}{\partial q}}\right] \qquad 5$$

From this expression it is clear that if $\partial x_i^*/\partial q$ is positive but less than x_i^*/q, then we cannot rule out the possibility that $\partial x_i^{**}/\partial w_i$ is non-negative. This says that in the case where an input price increase causes the equilibrium firm size to increase, the increase in firm size could overwhelm the pure substitution effect and cause the observed demand curve of the firm to be positively sloping. This, however, is not the end of competitive equilibrating process.

2. Output Market Equilibrium -- Variation in the Number of Firms

Next allow the number of firms to adjust so that price is equal to minimum average cost. Define the equilibrium as:

$$P(n^* q^*) \; = \; AC(w_1, \ldots, w_m, q^*) \qquad 6$$

where $P(.)$ is the Marshallian market demand curve. This equation is solved by allowing the number of firms to adjust to a value n^* that identifies the number of firms necessary to set price equal to minimum average cost. Each firm profit maximizes by choosing output so that marginal cost is equal to price. When there are n^* firms in the industry the market clearing price obtains when each firm operates at the output level associated with minimum average cost, q^*. The function $q^*(.)$ is solved from equation (2). Equation (6) implies a functional dependence among the set $\{n,w_1,...,w_m\}$ and can be solved for $n^*(w_1,...,w_m)$.

Differentiating equation (6) with respect to w_i allows us to solve for $\partial n^*/\partial w_i$.

$$\frac{\partial n^*}{\partial w_i} q^* P' + \frac{\partial q^*}{\partial w_i} n^* P' = \frac{\partial AC}{\partial w_i}$$

Rewriting,

$$\frac{\partial n^*}{\partial w_i} = \left(\frac{x_i^*}{q^{*2}}\right)\frac{1}{P'} - \frac{\partial q^*}{\partial w_i}\frac{n^*}{q^*} \qquad 7$$

where P' s the slope of the market demand curve. We see from this expression that the only way that the equilibrium number of firms can increase in the event of an input price increase is if the equilibrium firm size declines. If we rewrite (7) in elasticity terms it has a somewhat more intuitive interpretation:

$$\frac{\partial n^*}{\partial w_i}\frac{w_i}{n^*} = \left(\frac{x_i^* w_i}{Pq^*}\right)\frac{1}{P' \dfrac{n^* q^*}{P}} - \frac{\partial q^*}{\partial w_i}\frac{w_i}{q^*}$$

This says that if the percentage decline in the equilibrium firm size is larger than the percentage change in the market quantity demanded times the input's share of total cost (and revenue because they are equal), then the number of firms increases even though the market is shrinking.

We can now define the competitive equilibrium industry input demand, X_i^*, in terms of the equilibrium number of firms in the industry.

$$X_i^* = x_i^{***}(w_1,\ldots,w_m,P(n^*q^*)) = n^* x_i^{**}(w_1,\ldots,w_m) = n^* x_i^*(w_1,\ldots,w_m,q^*)$$

The slope of the industry input demand curve can be found by differentiating with respect to w_i.

$$\frac{\partial X_i^*}{\partial w_i} = n^*\frac{\partial x_i^{**}}{\partial w_i} + \frac{\partial n^*}{\partial w_i} x_i^{**}$$

For the industry input demand to be positively sloping, the firm-level demand must be positively sloping and overwhelm the decrease in the number of firms; (these two effects cannot both be positive as shown in eqt. 7). Substituting from (5) and (7) gives

$$\frac{\partial X_i^*}{\partial w_i} = n^* \frac{\partial x_i^*}{\partial w_i} + \left(\frac{x_i^*}{q}\right)^2 \frac{1}{P'} - n^* \left[\frac{\frac{x_i^*}{q} - \frac{\partial x_i^*}{\partial q}}{\frac{\partial MC}{\partial q}}\right]^2$$

Since all terms in this last expression are negative, the industry input demand curve is unambiguously downward sloping. Notably, because the term [x_i^*/q^* - $\partial x_i^*/\partial q$] is squared, the slope of industry input demand does not depend on whether the input is superior, normal, or inferior.

3. An Addendum on Short-Run Industry Input Demand

The form of the analysis used above can be usefully employed to analyze the short-run industry input demand. Consider the case where price adjusts to the new market equilibrium determined by the marginal cost curves of the existing firms and no new firms enter the market.[3] The equation describing this market equilibrium is

$$P(\bar{n}\hat{q}) \;=\; MC(w_1,\ldots,w_m,\hat{q}) \qquad 8$$

where $\bar{n}$ is the number of firms necessary to achieve the zero profit competitive equilibrium in the state prior to the input price change. That is, we start the comparative static analysis at a point of competitive equilibrium so that $\bar{n} = n^*$. Similarly, from the start $\hat{q} = q^*$, but $\hat{q}$ is the value of q necessary to solve equation (8). Like q^*, $\hat{q}$ is a function of input prices.

Differentiating with respect to w_i we get

$$\bar{n}P'\frac{\partial \hat{q}}{\partial w_i} = \frac{\partial MC}{\partial q}\frac{\partial \hat{q}}{\partial w_i} + \frac{\partial MC}{\partial w_i}$$

which can be rewritten as

$$\frac{\partial \hat{q}}{\partial w_i} = \frac{\frac{\partial x_i^*}{\partial q}}{\bar{n}P' - \frac{\partial MC}{\partial q}} \qquad 9$$

We can write the firm's demand for the *i*th input to include $\hat{q}(.)$

$$\hat{x}_i^* = x_i^*(w_1,\ldots,w_m,\hat{q})$$

The derivative of this function is then

$$\frac{\partial \hat{x}_i^*}{\partial w_i} = \frac{\partial x_i^*}{\partial w_i} + \frac{\partial x_i^*}{\partial q}\frac{\partial \hat{q}}{\partial w_i}$$

[3] The marginal cost curves are again defined by the optimal input levels given by equation (1).

Substituting from equation (9) above gives

$$\frac{\partial \hat{x}_i^*}{\partial w_i} = \frac{\partial x_i^*}{\partial w_i} + \frac{\left(\frac{\partial x_i^*}{\partial q}\right)^2}{\bar{n}P' - \frac{\partial MC}{\partial q}}$$

Marginal cost must be positively sloped in order to satisfy the sufficient second order conditions for profit maximization. That and downward sloping demand make the denominator of the last term negative. The numerator is squared so it is positive even if the partial itself is not signable. The first term is the basic, output constant demand, which is always negative. Thus, short-run industry demand for an input must be downward sloping.

References

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