# The Coase Theorem and Other Multimarket Equilibria

## Drug Prices

Consider the utility function for two goods, *x* and *y*:

$$U = \alpha x + xy + \beta y$$

This function generates demand functions of the form:

$$x = \frac{B}{2P_x} + \alpha \frac{P_y}{2P_x} - \frac{\beta}{2} \qquad 1$$

and

$$y = \frac{B}{2P_y} + \beta \frac{P_x}{2P_y} - \frac{\alpha}{2} \qquad 2$$

where $P_i$ are prices and *B* is the budget constraint. The parameters α and β measure the utility effect of good *x* relative to good *y*. Both parameters are assumed to be positive.

Consider the meaning of α and β. These are substitutability parameters between *x* and *y*. As α grows, *x* becomes more substitutable for *y*. As α grows, the demand for *x* grows. The effect is similar to the effect of income. That is, as α grows, expenditures on *x* grow. At the same time, as α grows, the *quantity* of *y* declines.

Assume that the two goods are produced by separate firms holding patents to their production so that the market supply is monopolized to each separately. However, the fact that they are substitutes in the consumer's utility function means that the two markets simultaneously equilibrate to determine price.

Assuming constant marginal cost, *m*, we can specify the profit functions of the two firms as:

$$\pi_x = P_x x - mx = (P_x - m) \cdot \left[ \frac{B}{2P_x} + \alpha \frac{P_y}{2P_x} - \frac{\beta}{2} \right]$$

and

$$\pi_y = P_y y - my = (P_y - m) \cdot \left[ \frac{B}{2P_y} + \beta \frac{P_x}{2P_y} - \frac{\alpha}{2} \right]$$

The first order conditions are:

$$\pi'_x = -\beta P_x^2 + mB + \alpha m P_y = 0 \qquad 3$$

$$\pi'_y = -\alpha P_y^2 + mB + \beta m P_x = 0 \qquad 4$$

The bi-market equilibrium is defined by solving the two first-order-condition behavior functions simultaneously.

Several things are notable. First, if the parameters α and β both equal one, the demands for the two goods are symmetric and the markets equilibrate to the same price for both goods. Second, as α increases the bi-market equilibrium price of *x* increases while *y* falls. This can be seen by differentiating equations (3) and (4) with respect to α. The matrix solution looks like this:

$$\begin{bmatrix} -2\beta P_x & \alpha m \\ \beta m & -2\alpha P_y \end{bmatrix} \begin{bmatrix} \dfrac{\partial P_x}{\partial \alpha} \\ \dfrac{\partial P_y}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} -mP_y \\ P_y^2 \end{bmatrix}$$

which gives the comparative static results:

$$\frac{\partial P_x}{\partial \alpha} = \frac{mP_y^2}{\beta(4P_xP_y - m^2)} \qquad 5$$

$$\frac{\partial P_y}{\partial \alpha} = \frac{-P_y(2P_xP_y - m^2)}{\alpha(4P_xP_y - m^2)} \qquad 6$$

Since price must be larger than marginal cost for the market to exist, the denominators of equations (5) and (6) are positive. Hence, the change in the price of good *x* shown in equation (5) is positive and from equation (6) the change in the price of good *y* is negative with respect to a change in α.

If we take as the starting point the condition that both α and β are equal to one and that prices are equal across the markets, the rate of increase in $P_x$ as α increases is less than the rate of decrease in $P_y$. Again since marginal cost cannot be greater than price, equation (6) is greater in absolute value than equation (5) implying that the price of good *y* goes down by more than the price of good *x* goes up as α increases. An implication of this result is to consider what happens as α and β both increase holding α equal to β. As long as α=β, $P_x$=$P_y$. As both α and β grow together, prices of both goods fall. In the limit as α and β become arbitrarily large, prices converge to marginal cost. This means that as both goods become more substitutable for each other, that is, both α and β grow, the prices of both goods fall.

The interpretation of the model in the context of our drug pricing problem is this. The more substitutable is one drug for another, the higher its price will be. That is, as α increases holding β constant, good *x* is increasingly demanded as a substitute for good *y*. Because of this the equilibrium price of good *x* increases as α increases holding β constant. We can think of *x* as a composite good and α can be considered the number of goods in this composite. Hence, the model says that as the number of substitutes for good *y* increases, the equilibrium price of good *y* falls.

## The Coase Theorem[1]

First consider the following question:

The Endangered Species Act (ESA) regulates (to the point of prohibiting) the development of land that is known to be a habitat for endangered species. This makes it profitable for landowners to purge their land of any such species prior to their discovery on the parcel of land to be developed. (This practice is sometimes known as "shoot, shovel, and shut up.") Recently the journalist Gregg Easterbrook proposed a modification of the ESA that would preserve wildlife habitat without the aforementioned incentives of the current law. His proposal is that the federal government charge *all* developers (not simply those who have endangered species on their property) a $1,000 per acre fee to be paid in conjunction with building permits. The proceeds of this fee would be used to purchase land best suited for wildlife habitats.

An environmental economist has commented on the Easterbrook proposal. Excerpts from his comments are reprinted below, with numbers added for purposes of this question:

[1] "What is good [about Easterbrook's proposal]... is that it removes the incentive for an owner to get rid of listed species."

[2] "What is bad is that Easterbrook does not recommend respect for property rights. Rather, he is offering a plan that would allow landowners to buy out of some land regulations. Any landowner, by paying $1000 per acre, could win control of the land that he or she 'owns.'"

[3] "The plan might be legitimate if it replaced taking from the few with a general tax on the many. But Easterbrook still proposes that all the cost be borne by landowners who develop their land and those who buy it....But why should those who want to build be required to carry a burden that all society should carry if all society truly benefits from protection of these species"

[4] "Furthermore, the proposal leaves in bureaucrats' hands most of the decisions about protecting species (which species should have priority? Which populations? What level of protection?) A true free market solution would leave these decisions in the hands of individuals and voluntary associations...Instead of political fights, we...want private individuals and associations to protect habitat and species in the ways they know best. We want Nature Conservancy preserves, Audubon Society refuges, local land trusts, and voluntary groups that put up bluebird and purple martin nesting boxes. Let's not turn them over to a federal bureaucracy."

The Coase Theorem has come to mean a number of things: (1) Externalities are symmetric. When one economic agent imposes costs on another it is not the "fault" of the one any more than the other. In regard to the question on the Endangered Species Act posed above, when development threatens wildlife with extinction, the problem exists both because some consumers demand new homes and shopping centers as well as because some individuals value the continued survival of earth species. There is no "right" or "wrong."

(2) When the transactions costs are low, the optimal allocation of resources will be achieved without government intervention. And, (3) if transactions costs are low, government intervention that takes the form of assigning property rights will have no effect on the allocation of resources.

[1] Varian (1992) Ch 24; Silberberg (1990) Ch. 17.8; Layard and Walters (1978) Sec. 6.3, 192-194, Sec 7.2, pp 224 - 227, Sec. 1.4, pp 22-26, Nicholson (1998) ch 24 p 719-41. See also, Ronald Coase, "The Problem of Social Costs," *JLE*, Oct 60, 1-44.

The theorem has generally been applied to negative externality situations, situations in which the production by one economic agent negatively affects the production of another. After decades of debate most commentators now hold that in cases where the two parties can negotiate an agreement, the so-called spillover effect is internalized at the firm level and at the market level as well. The result of this analysis is to consider the two parties to act as if they were one joint profit maximizer.

There are several ways that this might be modeled; consider the following. Let the profits of the two economic agents be stated separately. Profits for the externality generator are given by

$$\pi_1 = P_1 q_1 - C^1(q_1) \qquad 7$$

while profits in the recipient look like:

$$\pi_2 = P_2 q_2 - C^2(q_1, q_2) \qquad 8$$

Eqt. (2) captures the externality in the cost function. The partial of $C^2$ w.r.t. $q_1$ defines the externality. If the partial is positive, costs at firm 2 rise with production at firm 1. This is a negative spillover. A negative partial implies a positive externality.

The FOC for joint profit maximization are given by

$$P_1 = \frac{\partial C^1}{\partial q_1} + \frac{\partial C^2}{\partial q_1} \qquad 9$$

and

$$P_2 = \frac{\partial C^2}{\partial q_2} \qquad 10$$

Eqt. (3) shows that optimally the externality generator takes account of its effect on the profitability of the recipient. In the case where the partial effect of $q_1$ on the $C^2$ is positive, the output of the polluter is smaller than it would otherwise be.

The SSOC are defined by the Hessian:

$$H = \begin{vmatrix} (-C^1_{11} - C^2_{11}) & -C^2_{12} \\ -C^2_{12} & -C^2_{22} \end{vmatrix}$$

The SSOC require that marginal cost in good two be increasing and that the sum of the effect on marginal cost in goods one and two w.r.t. an increase in good one be increasing. Note that if the externality generator were to operate in isolation, profit maximization would require that marginal cost be increasing.

The comparative statics of the model are straightforward. An increase in own price causes output to increase. An increase in $P_1$ will cause output of good two to change based on the cross partial of the cost of good two. That is,

$$\frac{\partial q_2}{\partial P_1} = \frac{-C_{21}^2}{H}$$

We imagine that in the normal case the externality has the same effect on marginal cost that it has on total cost. Hence, if $C_1^2$ is positive, so too is $C_{21}^2$. Hence, output of good two declines as the price of good one increases.

Consider the graph labeled Figure 1. It shows the total profits associated with the externality situation described by the model. Total profits, $\pi$, associated with firm 1 are measured upside down. Measured right side up are the profits associated with firm 2. Profits associated with firm 2 are graphed as a function of $q_1$ only. That is, even though cost and profits for firm two are a function of both outputs, we are only representing the relation w.r.t. the output of firm 1 in the graph. Profits of firm 2 are negatively related to output level of firm 1.

The axes of the two pictures are compressed until the two profit functions touch. At this point of tangency, the marginal condition of joint profit maximization w.r.t. output one is satisfied. The slope of $\pi_1$ at that point is $[P_1\text{-}C_1^1]$. The slope of $\pi_2$ is $C_1^2$. The FOC require that these values be equal and they are at the tangency.

Isolated, suboptimal behavior is shown in the graph in two different ways. In one case, firm 1 produces without regard to its effects on firm 2. This is given by output labeled $\hat{q}_1$. Clearly at this point, given the way the graph is drawn, the combined profits of the two firms are lower. The other way in which noncooperative behavior is depicted is the profit level associated with zero firm 1 output. At the origin the profits of firm 2 are shown when $q_1 = 0$. These are also lower, as drawn, than the profits of joint maximization at $q_1$*.

The extra profits associated with joint profit maximization reflect point (2) of the Coase Theorem, that is, there are gains-from-trade in an externality situation. Bargaining potential between the affected parties can induce optimal behavior and will when transactions costs are low enough. However, the last part of the Coase Theorem says that the assignment of property rights does not matter. The fact that point (2) of the theorem is true gives us confidence in point (3). Even so, there are aspects of the picture shown in Figure 1 that give us pause.

It is immediately apparent from Figure 1 that the assignment of property rights does matter in the sense that whichever firm gets the property right, it is better off than if it didn't. While the bargaining solution in terms of output is still the same, the level of profits varies substantially. This leads us to question the effect of the level of profits on production. The level of profits affects production in the neoclassical model by way of entry and exit of firms. Entry of firms drives output price down; exit causes price to increase.

Competitive adjustment calls into question the many possible relations that could exist within the framework depicted in Figure 1. Under some circumstances, the distance labeled $\pi_2(q_1=0)$ could be larger than the distance labeled *ac*, which is the level of profit associated with joint profit maximization. What is the interpretation of such a picture? It is that isolated production of good 2 is more valuable than the joint production of goods one and two. This begs the question, why, if there is a negative relation between the two processes, would there ever be joint production? The answer is because locations are scarce. Location is the missing market. Pollution is only a problem when and if there are too few sites to locate the offensive and offended producers so that they operate in isolation. Joint production only occurs when the common use of a site is at least as profitable as isolated production. In this sense, the profit levels of the producers are all land rents. The profits described flowing to one producer or the other are merely residuals of operating revenues that are paid out to the owner of the location on which production operates.

Now it is clear that the definition of property rights does not matter. If the court rules for one or the other party in a nuisance case, operation is not affected. The land that houses the joint activities is consolidated and the landlord of the total property determines the production mix that maximizes the total value of the site.

In most of the cases that Coase himself mentioned, the level of the spillover was (or should have been) naturally negotiated by the landlord.[2] This piques our interest in thinking about the internalization of spillover relationships as a locational phenomenon. The most obvious case that comes to mind is the shopping mall.

In shopping malls there is a spillover from one store to another. The spillover that is most interesting is the positive effect that the *anchor stores* have on the *specialty shops*. The customers who come to shop at the anchor stores will sometimes shop at the specialty stores as well. As a consequence, the specialty shops receive some of the benefits of the advertising and brand name appeal of the anchor stores. This is the case of a positive externality. Modeled as above, the positive externality involves a relation in which the profits of firm 2 are enhanced by the production of firm 1. Figure 2 depicts the relation. Shown there we see the two firms operating at a common site. The production level of firm 1 causes the profits of firm two to be higher. As shown, the joint profits of the two are greater than the profits of firm 1 in isolation and the profits of firm 2 are negative in the absence of firm 1.

Let's consider a concrete case for sake of exposition. In Atlanta, there are two K-Marts operating at different locations. One operates in a strip mall, that is, in a shopping center next to a number of smaller stores. They all share a common parking lot and probably some other common services. At another location, K-Mart operates by itself. Cordon off the land mass associated with the strip mall. Consider the same land parcel at K-Mart's isolated production site. Figure 2 predicts that K-Mart's production at the isolated site is smaller than its production at the strip mall. That is the square footage of the K-Mart store is bigger in the shopping center. At the same time, the profits associated with K-Mart in the shopping center are lower. Assuming that all of the the profits at the two locations go to land rents, the rental rate in the shopping center has to be lower.

This makes sense inasmuch as the shopping center also houses stores that benefit from the traffic created by K-Mart. The shopping center developer puts together the package that maximizes land rents. The developer induces K-Mart to produce more, that is, to build a bigger store. However, sense the bigger store lowers profits, K-Mart must be subsidized to do this. The alternative is for K-Mart to operate in isolation, in which case it would make $\hat{a}\hat{b}$. Assuming that this would all be paid in rent on the isolated location, the rent subsidy that K-Mart receives in the strip mall is the difference between this and its profits there, *ab*.

[2] Of course, if the land is owned by different individuals, there is always the chance of changing the relative land values by inducing government to change the property right definition. For instance, there was the case some years ago of a quarry in Greenville that was expanding. The quarry was sited on some $x$ number of acres of which $y < x$ were actually quarried. The company was planning to increase $y$. Neighboring home owners objected claiming that the increased noise would lower their property values. I am not sure how the story ended but the Coase Theorem tells us it doesn't matter. If the landowners were successful in their bid to stop the expansion, and if the expansion was more valuable than the homes, the quarry company would just buy the homes, expand and then resell the homes.

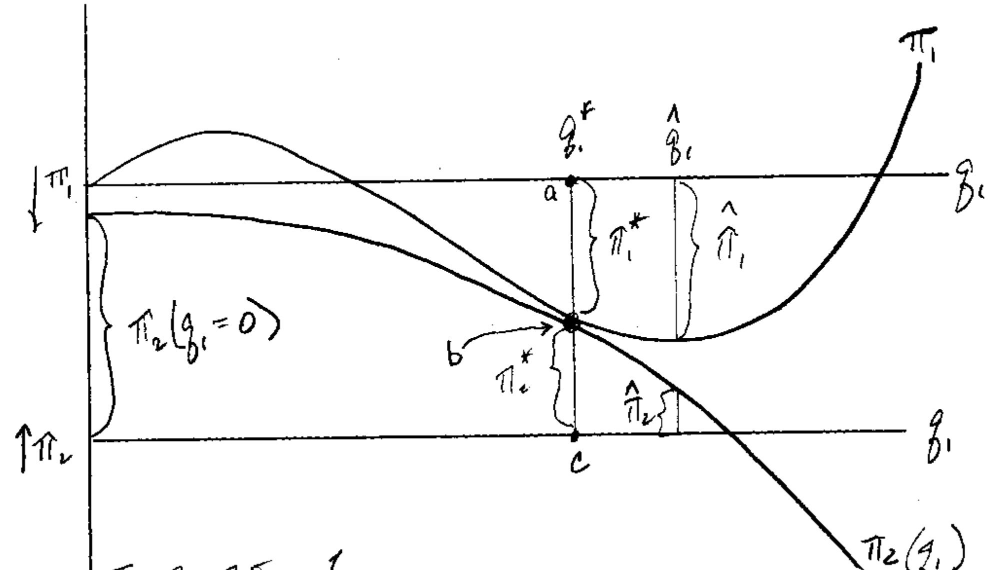


FIGURE 1

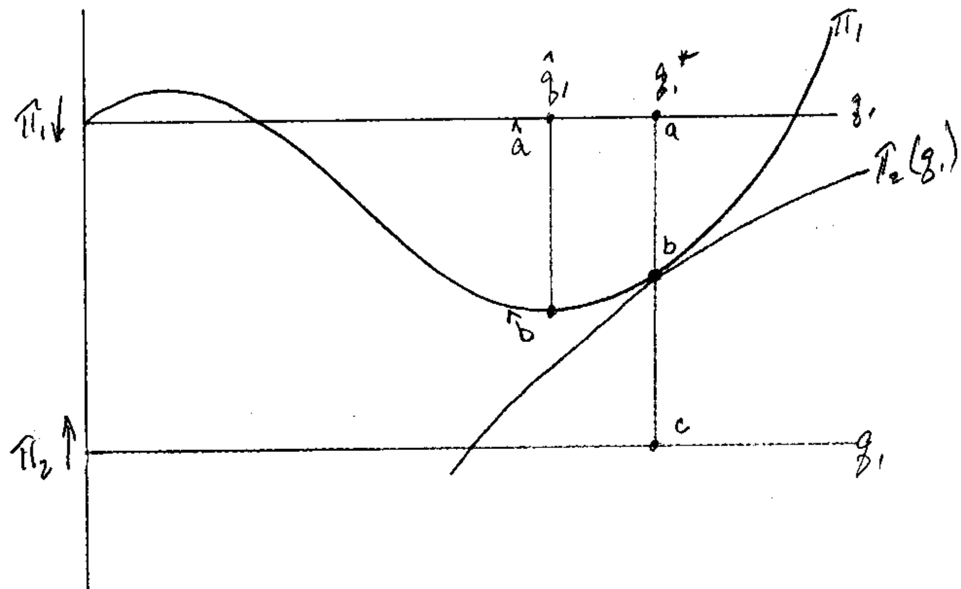


FIGURE 2