

EXPERIMENTING TO DEMONSTRATE THE LAW OF DEMAND¹

Standard Analysis

The model of consumer behavior assumes that the individual consumer maximizes a utility function subject to a budget constraint. The normal treatment usually starts with the discussion of a utility function.

This function has a number of properties which are carefully described. Basically the utility function orders all alternative combinations of two goods as they are viewed by the consumer. **INDIFFERENCE CURVES** link combinations that produce the same level of happiness (utility) for the consumer. Indifference curves cannot cross. They are downward sloping and they are convex to the origin.

The budget constraint is defined by a certain income per period of time. Goods have money prices. The budget constraint shows the maximum combinations of the goods that the consumer can purchase per period with the income. We write the budget constraint as:

$$M = p_x x + p_y y$$

where M is the income, x and y are the quantities of the two goods consumed per period by the individual, and p_x and p_y are the prices.

A commonly used utility function is $U = x^\alpha y^\beta$. This function is the familiar Cobb-Douglas form. This function conforms to the properties that economists assume about utility functions. If we assume that the consumer's utility function is characterized by the Cobb-Douglas function, then the consumer maximization problem can be stated as:

$$\max_{\{x,y\}} L = x^\alpha y^\beta + \lambda(M - p_x x - p_y y)$$

This states the consumer's utility maximization problem between two goods in the Lagrangian multiplier form.²

The FOC are:

$$\frac{\partial L}{\partial x} = \alpha x^{\alpha-1} y^\beta + \lambda p_x = 0$$

$$\frac{\partial L}{\partial y} = \beta x^\alpha y^{\beta-1} + \lambda p_y = 0$$

$$\frac{\partial L}{\partial \lambda} = M - p_x x - p_y y = 0$$

¹ Battalio, Kagel. "Demand Curves for Animal Consumers" *QJE* vol 96, 1981, p1-15; Battalio, Kagel, Kogut "Experimental Confirmation of the Existence Of Giffen Good", *AER* 1991, p961-978 vol 81.

² See Lecture 4 for a discussion of Lagrangian multipliers and constrained optimization.

And, the SSOC are defined by the hessian:

$$H = \begin{vmatrix} \alpha(\alpha-1)x^{\alpha-2}y^\beta & (\alpha-1)(\beta-1)x^{\alpha-1}y^{\beta-1} & -p_x \\ (\alpha-1)(\beta-1)x^{\alpha-1}y^{\beta-1} & \beta(\beta-1)x^\alpha y^{\beta-2} & -p_y \\ -p_x & -p_y & 0 \end{vmatrix}$$

H must be positive for a maximum. After expanding the hessian, it can be shown that the SSOC hold for all values of α and β .

Given that the SSOC hold, the FOC are predictions about consumer behavior. What do they tell us? We can find out by solving them simultaneously.

This is fairly simple. Notice that:

$$-\lambda = \frac{\alpha U}{p_x x} = \frac{\beta U}{p_y y}$$

Cancel the U terms, solve for $p_y y$ and substitute into the budget constraint. This gives:

$$x^* = \frac{\alpha}{\alpha + \beta} M p_x^{-1}$$

This is a demand function. The demand function, $y^*(\cdot)$, looks similar. These demand functions have constant, unitary elasticity in own price and income. The cross-price effect is zero. Notice that α must be positive for positive consumption of x , which we will assume. Similarly, β must be positive. The ratio $\alpha/(\alpha+\beta)$ is the budget share of x^* . Also notice that the optimal ratio of x^* to y^* is not a function of income, but only a function of the budget share parameters and the prices.

In summary, if consumers maximize utility and if their preferences are characterized by Cobb-Douglas utility functions, then they have downward sloping demands with zero cross price effects and constant, unitary own-price and income elasticities.

Consider another utility function: $U = x e^y$. Using this function, the Lagrangian objective function is:

$$\max_{\{x,y,\lambda\}} L = x e^y + \lambda(M - P_x x - P_y y)$$

The FOC are:

$$\begin{aligned} e^y - \lambda P_x &= 0 \\ x e^y - \lambda P_y &= 0 \\ M - P_x x - P_y y &= 0 \end{aligned}$$

The FOC solve directly to yield the consumer demand functions.

$$x^* = \frac{P_y}{P_x}$$

$$y^* = \frac{M}{P_y} - 1$$

The SSOC are given by:

$$H = \begin{vmatrix} 0 & e^y & -P_x \\ e^y & xe^y & -P_y \\ -P_x & -P_y & 0 \end{vmatrix}$$

which must be positive for a maximum. When the determinant is expanded it yields:

$$H = e^y P_x (2P_y - xP_x)$$

Substituting for x from the demand function, we have

$$H = e^y P_x P_y$$

which is obviously positive for all positive prices.

The demand functions are a bit peculiar but not terribly so. The comparative static implications of these functions are, first, that for both x and y the demand curves are downward sloping. Second, the demand for x is income indifferent. That is, the consumption of x does not vary with M . This is a bit odd. We usually think of goods being either normal or inferior in income, but not indifferent. Even so, if they can be either normal or inferior, there is no logical reason why a good cannot be income indifferent.³ For y , the cross price effect is zero.

An Empirical Structure

The standard analysis using specific utility function is not completely satisfying. This is not to say that it is useless. Indeed, it is quite valuable when used to study any number of problems, like for instance consumer surplus. However, it lacks something. To see what it lacks, let's step back and ask ourselves what we have just accomplished.

The specific utility functions have allowed us to derived demand functions. These demand curves have reasonable properties. However, they are idiosyncratic. That is, the demand functions described by the first utility function are not the same as the demand functions derived in the second case. In the first case, both goods have unitary income elasticities. In the second case, one good has a zero income elasticity.

³ For y , the oddity is that it can admit to a negative number. There are two ways to think about this. One is simply to constrain the values of y to positive numbers. The other is to suppose an endowment of y that allows for the consumer to sell some of it. This latter approach opens other issues.

At all events, the question is, How can we empirically implement this analysis? We cannot observe utility so we cannot know which set of demand functions is appropriate. Possibly we could collect data on consumption behavior and see whether it matched the predictions of either of these utility functions, and possibly we could collect enough data to search across all possible utility functions to find the one that is the best characterization. However, it would be very valuable to be able to generalize our theory in a way that showed us exactly how to perform an empirical experiment. This is exactly what we will do next.

Restatement of the Model

First let's restate the assumptions we discussed above. The Model of Consumer Behavior is based on the following:

1. Consumers have the objective of maximizing happiness or utility.
2. Consumers can rank all combinations of goods in terms of the utility that they generate.
3. More goods yield more utility (More is preferred to less).
4. Indifference curves, which identify combinations of goods that yield the same level of utility, are convex to the origin (diminishing marginal rate of substitution).
5. Consumers face a budget constraint comprised of a fixed income and fixed prices of the goods from which they choose.
6. In the simple model, there are only two goods.

Economics is the science of choice. Consumers are faced with choices and we try to predict how they will behave. For the most part, the first four assumptions concern the utility function which we can't observe, but they do set the stage for an empirical prediction. It is the last two assumptions that seem to be most crucial as test conditions for our empirical work.

To obtain general predictions, let's to look at the two-good problem in the graphical setting. Think of the consumer's behavior in the consumption space of $\{x, y\}$. Let the consumer optimally choose values x_a and y_a given some budget constraint. Now let the budget constraint rotate on this point.⁴ Let the price of x go up and the price of y go down so that $x_a \Delta p_x = -y_a \Delta p_y$. In other words the two prices change in opposite directions and each by an amount so that the price change in one times current consumption is equal to the price change in the other times the current consumption of it. This means that the consumer is not forced by the budget constraint to change consumption. Notice that the ratio of the change in the price of y to the change in the price of x is $[-x_a/y_a]$.

What happens? If the consumer is characterized by downward sloping convex indifference curves, then we can say unambiguously and in general, the consumer will buy less x and more y .

As we study the theory of consumer behavior in more detail, we will learn that the prediction embodied in this graphical demonstration is the true Law of Demand. The Law

⁴ Construction of the graph is left to the student.

of Demand does not say that consumers will buy more when the price of something falls. Rather it says that consumers will buy more of something if its price falls relative to everything else.

This is what is shown in the graph. It is general in that it is not tied to a particular utility function. Moreover, it is empirically tractable. That is, we can execute a test of the Law of Demand by conducting an experiment organized exactly like the graphical exposition.

Before we do the empirical exercise, let's show that the result is mathematically general. Let's assume that $U=U(x,y)$ is a utility function that conforms to our assumptions. We can write the general model of utility maximization as:

$$\max_{\{x,y\}} L = U(x,y) + \lambda(M - p_x x - p_y y)$$

The FOC are:

$$\begin{aligned} L_x &= U_x - \lambda p_x = 0 \\ L_y &= U_y - \lambda p_y = 0 \\ L_\lambda &= M - p_x x - p_y y = 0 \end{aligned}$$

and the SSOC are defined by the hessian of second partials which must be positive for a maximum.

The FOC and SSOC imply demand functions of the form:

$$\begin{aligned} x^* &= x^*(p_x, p_y, M) \\ y^* &= y^*(p_x, p_y, M) \end{aligned}$$

Now we want to shock the consumer's equilibrium by changing both prices. We can write this comparative static experiment in the following way:

$$\begin{bmatrix} U_{11} & U_{12} & -p_x \\ U_{12} & U_{22} & -p_y \\ -p_x & -p_y & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial x^*}{\partial \alpha} \\ \frac{\partial y^*}{\partial \alpha} \\ \frac{\partial \lambda^*}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} \lambda \cdot \left(\frac{\partial p_x}{\partial \alpha} \right) \\ \lambda \cdot \left(\frac{\partial p_y}{\partial \alpha} \right) \\ 0 \end{bmatrix}$$

where

$$x^* \cdot \left(\frac{\partial p_x}{\partial \alpha} \right) + y^* \cdot \left(\frac{\partial p_y}{\partial \alpha} \right) = 0$$

That is, we assume that there is an offsetting change in the prices of x and y around the consumer's optimal consumption. This is the mathematical expression for the graphical analysis that we described above.

$$\frac{\partial p_y / \partial \alpha}{\partial p_x / \partial \alpha} = \frac{dp_y}{dp_x} = -\frac{x^*}{y^*}$$

Let the price of x go up and the price of y go down. We want to know the slope of the demand curve for this offsetting or *compensating* change in the prices of x and y . We can write:

$$\frac{dx^*}{dp_x} = \frac{\frac{\partial x^*}{\partial \alpha}}{\frac{\partial p_x}{\partial \alpha}}$$

Thus the slope of the demand curve in this compensated setting depends on the value $\frac{\partial x^*}{\partial \alpha}$.

Solving by Cramer's Rule, we have:

$$\frac{\partial x^*}{\partial \alpha} = \frac{\begin{vmatrix} \lambda \cdot \left(\frac{\partial p_x}{\partial \alpha} \right) & U_{12} & -p_x \\ \lambda \cdot \left(\frac{\partial p_y}{\partial \alpha} \right) & U_{22} & -p_y \\ 0 & -p_y & 0 \end{vmatrix}}{\begin{vmatrix} U_{11} & U_{12} & -p_x \\ U_{12} & U_{22} & -p_y \\ -p_x & -p_y & 0 \end{vmatrix}} = \frac{\lambda p_y \cdot \left(p_x \cdot \frac{\partial p_y}{\partial \alpha} - p_y \cdot \frac{\partial p_x}{\partial \alpha} \right)}{H}$$

The sign of $\frac{\partial x^*}{\partial \alpha}$ is unambiguous. The price of x increases with α and the price of y decreases. Thus, the numerator is negative. And, the denominator must be positive by the SSOC. Thus, the compensated demand curve is downward sloping.

This shows that our graphical derivation is general. The only thing that we have assumed in this mathematical derivation is that the utility function is well behaved, with downward sloping, convex indifference curves. This is what is shown in the graph.

Using the Model in an Experiment

Economics is an old science, but the simple observation that when price goes up people buy less predates the science. It is ancient. Even so, there is a distinction between common sense and science. Specifically, economic science delves into the furthest reaches of the theory of consumer behavior and examines evidence to see how each and every prediction of the theory fares.

One relatively new method of study is experimenting with non-human, animal subjects. These subjects are in fact quite appealing for conducting a test of the model we have just developed. The model is based on the choice between two goods. To be most meaningful, we need to study the behavior of an individual for whom the choice between two goods is important. Animals are ideal subjects in this regard.

Laboratory rats lead a bland life. There is no cable TV or ski trips to Colorado. For the most part, they just scurry about in their cages, eat pellets and drink water. They do like flavored drinks, and when presented the option of working to obtain it, they are happy subjects.⁵ The interesting thing about animal experimentation is imagining how to design the experiment. In constructing the experiment, the researcher gains a more keen awareness of what the theory really says.

We can use the Skinner Box to construct a laboratory experiment that tests the theory of consumer behavior defined by the six assumptions listed above. The Skinner box contains dispensers with levers. The box has two dispensers that give out the two different goods. The rat has a fixed number of presses and gets a certain amount of juice for so many presses. The number of presses necessary to receive juice varies between flavors.

A rat is presented with the option of “buying” servings of these drinks. It buys the drinks by pressing a bar over the cup into which the drink is dispensed. The rat is given a certain total number of presses to be allocated between drinks. The researcher varies the number of presses necessary per serving of each drink.⁶ The total number of presses is like income. The presses necessary to receive a drink are the price of the drink. The rat is choosing between two goods, with a fixed budget.

Based on the theory of consumer behavior, we assume that the rat tries to make itself as happy as possible. The model says that the consumer will choose a combination of goods along the budget constraint, and this combination of goods is the utility maximizing set. The consumer’s choice is characterized by a tangency between an indifference curve and the budget line.

Law of Demand: *If relative prices change so that the consumer’s budget rotates on the consumption choices, the consumer will choose less of the more expensive good and more of the less expensive good.*

⁵ Animal researchers have found that rats are particularly fond of Root Beer. They also like Tom Collins Mix (alcohol free).

⁶ Actually, the experiments are usually conducted in such a way that the cup sizes of the dispensed goods change and the animal need only press once per dispensed cup. Researchers have found that the animals more quickly respond to the set of implied prices using this approach.

The model is quite easy to implement experimentally. The researcher sets up an initial experiment with a budget and prices and then records the rat’s choices. The researcher then changes the budget in a way that pivots the new budget on the initial choice.

The empirical observation of the Law of Demand is then set. If the rat chooses more of the more expensive good, the Law of Demand fails.

This experiment is shown in the following table. In the first trial with a total budget of 60 presses and prices of 10 and 5 for root beer and collins mix, respectively, the rat chose 4 cups of root beer and 4 cups of collins mix. In the second trial, the researcher changed the price of root beer to 5 presses and the price of collins mix to 10. The total budget was left unchanged. Notice that these offsetting prices allow the budget to rotate on the initial consumption choice. At the new prices, the rat could continue to consume the initial levels. However, we expect that with a lower price of root beer and a higher price of collins mix, the rat will choose more root beer and less collins mix. The Law of Demand is challenged by the behavior of the rat in the second trial.

Experiment I: Offsetting Price Changes					
Trials	Total presses allowed	Presses required for Root Beer	Presses required for Collins Mix	Consumption of Root Beer	Consumption of Collins Mix
First	60	10	5	4	4
Second	60	5	10	??	??

Alternative Experimental Designs

The experimental design actually employed by the researchers is not always the one described above. Consider the following table.

Experiment II: An Income-Compensated Price Change					
Trials	Total presses allowed	Presses required for Root Beer	Presses required for Collins Mix	Consumption of Root Beer	Consumption of Collins Mix
First	60	10	5	4	4
Second	40	5	5	??	??

Here in the second trial, the rat is given a different budget of presses to offset or *compensated for* the change in the press price of one good. The price of the other good is left unchanged. Notice that the experiment is identical to the first in that the budget rotates on the initial consumption choices. The same prediction holds. We expect the consumption of root beer to increase because its compensated price has fallen.

Lastly it is interesting to see how the consumer responds to an uncompensated change in price. Such an experiment is described below. In this case, the actual responses of the rats are given for all trials. It is the third trial that represents the uncompensated price change in root beer when compared to the initial trial. Notice that the rat behaves according to the Law of Demand as shown by the comparison of trial two to trial one.

Also notice that the rat exhibits a negative, uncompensated price response as shown by the comparison of trial three to trial one.

Experiment III: Compensated and Uncompensated Price Changes					
Trials	Total presses allowed	Presses required for Root Beer	Presses required for Collins Mix	Consumption of Root Beer	Consumption of Collins Mix
First	60	10	5	4	4
Second	40	5	5	5	3
Third	60	5	5	6	6

It is useful to classify the effects depicted in this last experiment.

- The **PRICE EFFECT** is the change in consumption caused by a price change holding everything else constant.
- The **INCOME EFFECT** is the change in consumption caused by a change in income holding everything else constant.
- The **SUBSTITUTION EFFECT** is the change in consumption caused by a change in price and an offset change in income. It is also the change in consumption associated with a change in relative prices, i.e., the simultaneously change in two prices by amounts that are opposite but equally weighted.
- The **CROSS PRICE EFFECT** is the change in consumption of one good caused by a change in the price of the other holding everything else constant.

Consider a graph of the experimental trials and responses.⁷

The notion of a cross price effect deserves some commentary. Note that the cross price effect can be defined in gross or net terms. The net effect is the change in the consumption of collins mix as the relative price of root beer changes. Relative price in this case means an income adjusted price change in root beer. While the price of collins mix does not change, by changing both the price of root beer and the budget, the relative price of collins mix has changed. In a two good world, which is the situation that this experiment simulates, the goods must be substitutes in consumption as their relative prices change. This is true because of the Law of Demand. If the relative price of root beer goes down, then in order for more root beer to be consumed, less collins mix must be consumed. This net cross price effect is positive in sign. We call this a *substitute* good. In a two good world, good must be net substitutes.

However, there is also a gross cross price effect. The gross cross effect is the change in the consumption of collins mix as the price of root beer changes, holding everything else constant. This is the change in consumption that occurs between trial 1 and trial 3. The consumption of both root beer and collins mix increase as a result of the decrease in the price of root beer. Thus, in gross price terms, collins mix is a complement to root beer. As the price of root beer goes down, collins mix consumption increases. The

⁷ The student should draw this graph.

cross price effect is negative; we call this a complement. There is no requirement, even in the two good world, that goods be gross price substitutes.

The Law of Demand does not dictate that as price goes down, consumption must increase. That is, the Law of Demand does not say that the price effect must be negative. Rather it says that as relative price goes down, consumption must increase. This is called the pure **SUBSTITUTION EFFECT**. Both the first and second experiments showed the pure substitution effect: as relative price changes in one direction, consumption consequently changes in the other direction. Relative price can be calculated either in terms of offsetting price changes or in terms of an income compensated price change.

As a practical matter, the substitution effect is usually so dominant that we do not have to control for relative price changes. That is, most of the time, when the price of a good alone goes down, consumption increases. However, recognize that such a change is not a relative price change, but rather an absolute price change. The consequent change in consumption is called simply the **PRICE EFFECT**.

The fact that the Law of Demand holds for either offsetting price changes in both goods or an income compensated price change in one good suggests a relation between the price, income, and substitution effects. This relation is called the **SLUTSKY EQUATION**. We will explore this relation in some depth. However, for the time being let's just state it.

Slutsky Equation:

$$\text{Price Effect} + \text{Consumption weighed Income Effect} = \text{Substitution Effect}$$

Questions

Here are some questions that you should ponder:

1. What are the possible objections to this experimental evidence of the Law of Demand?
2. Given the evidence obtained from animal experimentation III, estimate the demand curve for root beer.
3. Recast the experiment reported above in terms of the size of the amount dispensed. That is, let the rat receive some of good for each press. However, the amount that is dispensed is varied between trials. Start out with 60 presses. Allow the rat to receive 5 cc. of root beer for each press or 10 cc. of collins mix. Assume that the conclusion of the first trial results in 40 root beer presses and 20 collins mix. Now structure the second and third trials to test the Law of Demand.
4. Do we need to make the assumption that indifference curves are convex in order to test the Law of Demand (in either experiments I or II)?