THE COMPLETE EMPIRICAL IMPLICATIONS OF THE THEORY OF CONSUMER BEHAVIOR

The Law of Demand is largely an empirical fact of nature. In practice we have an idea about how to empirically estimate demand curves, and we know that empirically, demand curves always have a negative price effect. The Law of Demand can be derived graphically and this simple exposition is sufficient guidance to design a controlled experiment that tests the law in both its basic and even more sophisticated dimensions. In our controlled experiment, we first examine the substitution effect which says that when relative prices change in one direction, consumption changes in the other. We find this to be true. Also, we know from the graphical exposition of the theory that while a positive price effect is theoretically possible, it can only occur when there is a sufficiently large, negative income effect to offset the pure substitution effect.

Hence, we are left with the somewhat puzzling question of why we need to derive the theory of consumer behavior in more detail? If empirical demand curves obey the simple law of demand, what more can we learn from a complicated theory. The answer to this query is the purpose of this lecture.

THE SET-UP

The theory of consumer behavior is a set of predictions about observable phenomena. These predictions are derived from a model in which individuals are assumed to maximize utility subject to limited purchasing power. Since utility is unobservable, the challenge of the theory is derive empirically meaningful statements predicting consumer behavior.

To summarize the model we have a utility function $U(\bullet)$ that ranks all combinations of the set of n goods. The consumer's objective function is

$$\max_{\{x_i, i=1,n\}} U(x_1,\ldots,x_n) \;\; s.t. \;\; M^0 = \sum_{i=1}^{n} P_i x_i \qquad \textbf{(1)}$$

The first and sufficient second order conditions (FOC & SSOC) of this problem imply that consumer behavior will be predicted by equations of the following sort:[1]

$$x_i^{\,M} = x_i^{\,M}(P_1,\ldots,P_n,M^0) \qquad \textbf{(2)}$$

The problem with eqt (2) is that the model does not yield any direct predictions about consumer behavior. That is, comparative static analysis does not identify the signs of the partial derivatives of the demand functions defined by eqt (2).

This problem is remedied by examining the *dual* of the maximization problem. Every optimization problem has a dual or a mirror image. In this case maximization of utility subject to an expenditure constraint can be looked upon as minimization of expenditures subject to a utility constraint.

$$\min_{\{x_i\}} M = \sum_{i=1}^{n} P_i x_i \;\; s.t. \;\; U^0 = U(x_1,\ldots,x_n) \qquad \textbf{(3)}$$

[1] The superscripts refer to the parameter that forms the constraint in the optimization problem.

The FOC & SSOC imply solution functions for the model in the form of:

$$x_i^U = x_i^U(P_1,\ldots,P_n,U^0) \quad \textbf{(4)}$$

THE THEORETICAL RESULTS

It turns out that these functions have comparative static implications. That is, there are several derivatives that can be identified. First we can show that $\partial x_i^U/\partial P_i < 0$. In words, this says that consumption of a good increases when its price falls if the consumer is forced to move along a utility contour. By itself this is not an empirically meaningful statement because we cannot measure utility directly or determine whether a consumer is constrained to stay at a given level of utility. However, because the derivative is signable, we can put the result to good use.

In addition to the negative sign of the own price effect, we can show that the cross price effects are symmetrical in the utility-constant demand functions. That is, $\partial x_i^U/\partial P_j = \partial x_j^U/\partial P_i$. This result derives from Young's Theorem. Since the order of cross partial differentiation does not matter, the coefficient matrix in the comparative static analysis is symmetric. In the expenditure minimization problem, the vector of constants has only one term. Thus, the symmetry comes into play.

Now we apply the *Duality Theorem*. Duality means that every maximization problem can be looked at as a minimization problem. The duality theorem says that the solution to the maximization problem is identical to the solution of its minimization dual when the constraint to the maximization problem is appropriately defined. Consider the optimized value of expenditures in the problem defined by eqt (3). This can be written as

$$M^U = \sum_{i=1}^{n} P_i x_i^U + \mu^U\left[U^0 - U(x_1^U,\ldots,x_n^U)\right] = M^U(P_1,\ldots,P_n,U^0) \quad \textbf{(5)}$$

Eqt (5) is called the indirect expenditure function because it expresses the minimized expenditure level necessary to achieve U^o as a function of the parameters of the problem. The duality theorem says that by making $M^U(\bullet)$ the value of the income constraint in the maximization problem defined by eqt (1), then the optimized value of utility in that problem will be equal to U^o . That is,

$$M^0 = M^U(P_1,\ldots,P_n,U^0) \Leftrightarrow U^M(P_1,\ldots,P_n,M^0) = U^0 \quad \textbf{(6)}$$

which makes intuitive sense. But more importantly and more rigorously, the behavior functions for the x_i's are solved simultaneously for the primal and the dual. Simply put, this means that the levels of the solution values are the same. That is, $x_i^M = x_i^U$ as can be seen in the graph in the two goods case. But it also means that equations (2) and (4) are equal at that point.

$$x_i^M(P_1,\ldots,P_n,M^U(P_1,\ldots,P_n,U^0)) = x_i^U(P_1,\ldots,P_n,U^0) \quad \textbf{(7)}$$

Equation (2) is observable but not predictive. Equation (4) is predictive but not observable. Equation (7) combines the best of both worlds.

THE EMPIRICAL APPLICATIONS

Slutsky Equation

Eqt (7) allows us to derive empirically meaningful statements about consumer behavior. First is the Slutsky equation. Differentiate eqt. (7) with respect to P_i. The result looks familiar:

$$\frac{\partial x_i^{\ M}}{\partial P_i} + \frac{\partial x_i^{\ M}}{\partial M}\frac{\partial M^U}{\partial P_i} = \frac{\partial x_i^{\ U}}{\partial P_i} < 0 \qquad \textbf{(8)}$$

The price effect from the ordinary demand curve plus the income effect times a scalar is equal to the slope of the compensated demand curve. The envelope theorem helps define this scalar. It says that $\partial M^U/\partial P_i = x_i^U$, and by eqt (7) itself, $x_i^U = x_i^M$. Hence,

$$\frac{\partial x_i^{\ M}}{\partial P_i} + \frac{\partial x_i^{\ M}}{\partial M} x_i^{\ M} = \frac{\partial x_i^{\ U}}{\partial P_i} < 0 \qquad \textbf{(9)}$$

Eqt (9) is empirically meaningful because every term on the left-hand-side of the equals sign is observable. This is the behavior of individuals when money income and prices are accounted for. These are partial derivatives. We observe a change in consumption with respect to a price change holding income constant. Then we observe a change in consumption with respect to an income change holding prices constant. Eqt (9) tells us how to add up these effects.

***Result* 1.1:** The Slutsky equation shown in equation (9) says that the own-price effect plus the income effect weighted by the consumption level must be negative.

In elasticity terms the Slutsky aggregation of the price and income effects takes the form

$$\varepsilon_{ii}^M + \varepsilon_{iM} S_i = \varepsilon_{ii}^U < 0$$

where ε_{ii}^M is the percentage change in the consumption of the i^{th} good with respect to a change in the i^{th} price hold income constant, ε_{iM} is income elasticity, S_i is the budget share of the i^{th} good, and ε_{ii}^U is the own-price elasticity along the compensated demand curve.[2]

***Result* 1.2:** In elasticity form, the Slutsky equation says that the sum of the own-price elasticity plus income elasticity times budget share must be negative.

The empirical nature of this hypothesis is revealed when we apply the theorem to estimated values of the price and income effects. Commonly, the price and income effects are estimated using a linear structure for the demand equation:

$$x_i = a + bP_i + cM + dP_j$$

[2] Multiply each term in equation (9) by P_i/x_i, and the second term on the left by M/M.

Linear regression returns coefficient values for $\partial x_i^M/\partial P_i$, which is labeled *b* above, and for $\partial x_i^M/\partial M$, which is *c*. The Slutsky equation says that -*b* must be bigger than *c* times the level of consumption. If it is not, the demand curve is misestimated (or the theory is wrong). Furthermore, the Slutsky equation tells us a consumption range over which the estimated demand curve applies.

***Result* 1.3:** Linear demand curves are only consistent with utility maximizing consumer behavior in the range of $x_i < -b/c$.

Specifically this means that linear demand curves may not be theoretically consistent for low price and high quantity combinations.

These same kinds of statements can be made with regard to estimated elasticities. Elasticities can be computed from linear demand curves or estimated directly using a log transformation. Either way Result 1.1 applies.

***Result* 1.4:** Demand curves are only valid in the range where budget share of the i^{th} good is less than $-\varepsilon_{ii}/\varepsilon_{iM}$.

In practice this means that log-linear demand curves are theoretically invalid at super high price levels when demand is inelastic and at super low price levels when demand is elastic.

Cross Price Effects

The cross price effects for the utility constant demand curves are symmetric. That is,

$$\frac{\partial x_i^U}{\partial P_j} = \frac{\partial x_j^U}{\partial P_i}$$

The nature of this result can be seen clearly in the graph of the two-good world. As the price ratio between the two goods changes and consumption slides along the indifference curve, the change in the price ratio can be viewed as either a change in the price of one good or the other. Hence, the change in consumption of *i* caused by a change in the price of *j* must equal the change in the consumption of *j* with respect to a change in the price of *i*.

We can empirically exploit this symmetry result by looking at the ordinary demand curves. The Slutsky equation is the own price differentiation of eqt (7). The cross price differentiation has a very similar form.

$$\frac{\partial x_i^M}{\partial P_j} + \frac{\partial x_i^M}{\partial M}\frac{\partial M^U}{\partial P_j} = \frac{\partial x_i^U}{\partial P_j} \quad \textbf{(10)}$$

Substituting based on the envelope theorem gives

$$\frac{\partial x_i^M}{\partial P_j} + \frac{\partial x_i^M}{\partial M} x_j^M = \frac{\partial x_i^U}{\partial P_j} \quad \textbf{(11)}$$

Based on the symmetry of the utility-constant demand curves, we can write:

$$\frac{\partial x_i^{\ M}}{\partial P_j} + \frac{\partial x_i^{\ M}}{\partial M} x_j^{\ M} = \frac{\partial x_i^U}{\partial P_j} = \frac{\partial x_j^U}{\partial P_i} = \frac{\partial x_j^{\ M}}{\partial P_i} + \frac{\partial x_j^{\ M}}{\partial M} x_i^{\ M} \tag{12}$$

or in elasticity form[3]

$$\frac{\varepsilon_{ij}}{S_j} + \varepsilon_{iM} = \frac{\varepsilon_{ji}}{S_i} + \varepsilon_{jM} \tag{13}$$

Eqt (13) shows an interesting anomaly in the theory. The observable substitutability or complementarity of x_i for x_j when looking at the demand for good *i* can reverse when we examine the demand for good *j*. Even though the utility constant cross effects are symmetric, observed behavior is conditioned by the *income effect* that is associated with any price change. So, for instance for the Chinese, fish is a complement to meat, but meat is a substitute for fish. That is, when the price of meat goes up, Chinese people eat less fish. However, when the price of fish goes up, they eat more meat. Seems odd, even for the Chinese.

***Result* 2.1:** The observed cross-price effects between two goods found in ordinary demand functions need not work in the same direction across demand functions.

Not only are the observed effects not reciprocal, as they are in the utility-constant demand relations, they can be of opposite sign. Good *i* can be a complement to *j*, while good *j* is a substitute for *i*. However we can note the following from eqt. (13): If $\varepsilon_{ij} < 0$ and $\varepsilon_{ji} > 0$, then $\varepsilon_{iM} > \varepsilon_{jM}$. Thus we have an empirical statement about income elasticities relative to cross-price effects.

***Result* 2.2:** If the cross-price elasticity of *i* for *j* is negative while *j* for *i* is positive, then good *i* (the complement to *j*) must have a higher income elasticity.

For the case of the Chinese consumption of meat and fish, the theory says that because of the signs of the gross cross price elasticities, the income elasticity of fish must be larger than the income elasticity of meat.

There are a couple of other things that we can point out looking at this form.

***Result* 2.3:** If the cross-price effects between two goods are equal, then the income elasticities must be equal.

Note that the statement is about the linear price *effects* compared to the income *elasticities*.[4] The result is derived from eqt. 12. The cross-price terms subtract out. Cross multiplying the

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[3] Multiply both sides of eqt. (12) by P_i/x_j and by P_j/x_i which gets the cross-price elasticity terms. Multiply the income effect terms by *M/M* to get the income elasticities. Cross multiply the left over x_i and x_j terms and divide both sides by *M*. This gives a budget share multiplier on both sides which can be used to cancel the budget share term next to the income elasticities.

[4] Be careful here to distinguish between the terms called “effects” and the elasticity expressions. The partial of x_i with respect to p_j is not the same thing as the elasticity of good *i* with respect to price *j*. Elasticity is a weighted “effect”.

consumption level terms and dividing both sides by M gives $\varepsilon_{iM} = \varepsilon_{jM}$. Thus, whether the goods are substitutes or complements, the goods must have identical income elasticities if they have reciprocal cross-price effects.

***Result* 2.4:** If the income elasticities are equal, then the cross-price effects between two goods must be equal.

This is also most easily seen in eqt. (12). Remember to note the difference between elasticities and effects.

***Result* 2.5:** Inferior goods will have relatively large cross-price elasticities in real numbers.

This says that inferior goods will be less complementary if both goods are complements and more substitutable if the cross price elasticity is positive. There must be more substitution potential to make up for the negative income effect. If $\varepsilon_{iM} < 0$ in eqt (13), then ε_{ij} has to take up the slack when good i is paired with substitute goods that have normal income effects.

Homogeneity

Because income-constant demand curves are homogeneous w.r.t. income and prices, we can employ Euler's Theorem to make another empirical statement. We can show that observable demand curves are homogeneous of degree zero by deriving the FOC and SSOC at one set of prices and income and comparing these the FOC and SSOC derived at another set that differs by a factor of proportion k. If the FOC and SSOC of the two problems are identical, then the solution equations are homogeneous of degree zero in money and prices.[5]

Euler's theorem says that the sum of the partials times the levels of the variables equals the degree of homogeneity. For demand curves this means

$$\sum_{i=1,\ldots,i,\ldots n}\left[\frac{\partial x_i^{\,M}}{\partial P_i}P_i\right]+\frac{\partial x_i^{\,M}}{\partial M}M=0 \qquad \textbf{(14)}$$

or in elasticity form

$$\sum_{i=1,i,n}\varepsilon_{ij}+\varepsilon_{iM}=0 \qquad \textbf{(15)}$$

The empirical value of eqt (15) allows for a specification test of estimated demand curves.[6] Note that eqt (15) is a restriction placed on a single demand curve. It applies to the demand for one good with respect to all of the arguments in that demand function. Since the sum of the elasticities is zero when all relevant price variables are included, the observed sum of estimated elasticities is a specification test.

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[5]You should be able to derive this result.

[6] Note that the result given by eqt (15) is the same as the result that the sum of the price elasticities for a compensated demand curve must sum to zero.

***Result* 3.1:** The sum of own-price, cross-price, and income elasticities within a demand function must sum to zero.

Empirically this means that the more the sum of own-price, cross-price, and income elasticities differs from zero, the greater is the bias of omitted variables. In order to explore this result, let's return to the rat experiment results. In that exercise we were able to estimate a price effect and an income effect. If we look at one point on the demand curve and evaluate eqt. (14), we can form an estimate of the cross price effect. For instance, if we look at the demand curve related to an income of 60 presses, at the price of 5 presses for root beer, the own price effect times price is -2 and the income effect times income is +3, hence the cross price effect times the price of Collins Mix must be -1 for the homogeneity condition to hold. Also, this implies that the intercept value of the linear demand form must be 6.

The homogeneity result applied to empirical demand curves presents somewhat of a paradox. Notice that if we had evaluated the demand curve at the price of 10 presses for root beer, the weighted own-price effect would have been -4. Since the weighted income effect is still +3, for the homogeneity condition to hold at that point, the weighted cross price effect would have to be +1. Since the price of Collins Mix has not changed, the implication is that the cross price effect changed. This is inconsistent with a linear specification of all effects.

Paradoxical or not, this represents a limitation of the linear estimating form. That is, using a linear estimating form for a single demand curve, the homogeneity condition cannot be imposed as an estimation constraint for every observation. Nonetheless, the homogeneity condition can be tested for and imposed as a constraint at the mean of the data on the independent variables.

Consider a estimating form of the sort:

$$Q_1 = \alpha + \beta_1 P_1 + \beta_2 P_2 + \gamma M$$

If the true demand curve is $Q_1(P_1, P_2, P_3, M)$ then in our estimated $\hat{\alpha} = \alpha + \beta_3 \overline{P_3}$ if estimated in unbiased fashion. However, we expect that it will be biased as will the estimates of the other parameters because the prices are likely to be correlated. Even so, if $[\beta_1 \overline{P_1} + \beta_2 \overline{P_2} + \gamma \overline{M}] / Q_1 = 0$, then the estimated intercept term is an unbiased estimate of the true α, which is the mid-point of the consumer's choices in linear form.

***Result* 3.2:** If two markets are related only by the cross price effects in their demands and they are substitutes, then the markets will have a well defined joint equilibrium.

This result is the point of the study question below. It is most easily seen where we assume there is only one other price in the demand curve and that other good is a substitute. Eqt. (14) tells us that the weighted own-price effect, which is naturally negative, must be as large as the sum of the weighted cross price effect and income effect. A sufficient condition for the joint equilibrium is that the product of the own-price effects be bigger than the product of the cross price effects. This will be the case based on eqt. (14) above. Even though the price effects in eqt. (14) are weighted by the prices, the algebra works out.

Consider a simple demand and supply model in two related markets:

$$Q^D{}_1 = a\, I - b\, P_1 + c\, P_2 \qquad Q^D{}_2 = f\, I - g\, P_2 + h\, P_1$$

$$Q^S{}_1 = d + e\, P_1 \qquad Q^S{}_2 = i + j\, P_2$$

P is price, *Q* is quantity and *I* is income. The coefficients are signed explicitly. The equilibrium condition prevails: quantity demanded equals quantity supplied in both markets.

a) What is the effect of a supply shift in market 1? That is, derive the comparative static results on price in both markets with respect to a parametric change in the supply condition in market 1.
b) What conditions must be imposed to get a determinate solution?
c) Are these conditions consistent with the theory of consumer behavior?

Homogeneity can also be applied to the compensated or utility-constant demand curves. Utility constant demand curves are homogeneous of degree zero in prices. That is, if all prices increase by a constant factor, the slope of the budget constraint does not change and the optimal consumption bundle remains the same. Formally, we derive the result using Euler's Theorem:

$$\sum \frac{\partial x_i^U}{\partial P_j} P_j = 0$$

Because we know that the own price effect, $\frac{\partial x_i^U}{\partial P_i}$, is negative, the rest of the terms must sum to a positive number. Hence,

***Result* 3.3:** Every good has at least one net substitute.

Budget Constraint[7]

One last set of empirical statements can be derived by substituting the demand curves into the budget constraint and differentiating. In elasticity form, differentiating with respect to *M* yields

$$\sum_{i=1}^{n} \varepsilon_{iM} S_i = 1 \qquad \textbf{(16)}$$

and with respect to P_j yields

$$\sum_{i=1}^{n} \varepsilon_{ij} S_i = -S_j \qquad \textbf{(17)}$$

Eqts (16) and (17) are restrictions on the estimation of systems of demand functions. These are implications about income and price elasticities across demands.

[7] Silberberg (1990)&(2001)sections 10.5-10.6; Intriligatar, Bodkin, Hsiao (1996)-Chapter 7-sections 7.1-7.2. Layard and Walters (1978) section 5.2. Nicholson (1989) Chapter 7; Nicholson (2000) Chapter 4.

Eqt. (16) tells us what must happen if income goes up. If income goes up, the income elasticities identify how consumption of each good changes. However, we know that the budget constraint has to be satisfied overall. Equation (16) says,

***Result* 4.1:** The sum of the budget share weighted income elasticities must sum to one.

Equation (17) speaks to the effect of a change in one price across demand functions. It is best seen when considering the two-good case. Let there be good 1 and everything else, good 0. Rewriting eqt. (17) gives:

$$\varepsilon_{01} = -\frac{S_1}{1-S_1}(\varepsilon_{11}+1) \qquad \textbf{(18)}$$

This shows that the change in the consumption of everything else when the price of good 1 changes is a function of the price elasticity of good 1. This idea can be summarized as follows:

***Result* 4.2:** For elastic goods, substitutes dominate complements; for inelastic goods, complements dominate substitutes.

Consider the case where good 1 is price elastic. When the price of good 1 increases total spending on good 1 falls. This extra money must be spent elsewhere. In the two good world, it is spent on the composite commodity. Hence, in the demand function for the composite commodity, when the price of good one increases, consumption of the composite commodity increases. The situation is reversed when good 1 is price inelastic.

More elaborately, let there be four goods, beer, wine, peanuts, and everything else. Now let beer be price elastic, peanuts be a complement to beer, wine a substitute, and let the budget shares of beer, wine, and peanuts all be the same. What can we say about the relative purchases of wine and peanuts? Rewriting eqt (17) we have

$$\varepsilon_{21}\frac{S_2}{S_1} + \varepsilon_{31}\frac{S_3}{S_1} + \varepsilon_{41}\frac{S_4}{S_1} = -(\varepsilon_{11}+1) > 0 \text{ where } \varepsilon_{11} < -1 \qquad \textbf{(19)}$$

for beer defined as good 1, peanuts as 2, wine as 3, and the rest as 4. Let the effect of a change in the price of beer be confined to only wine and peanuts. That is, let e_{41} be zero. To satisfy equation (19), wine expenditures (the substitute) must react more than peanut purchases (the complement) as the price of beer changes.

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GLOSSARY OF TERMS AND RESULTS

$\frac{\partial x_1{}^M}{\partial P_1}$	Own-price effect--slope of ordinary demand curve
$\frac{\partial x_1{}^M}{\partial M}$	Income effect--shift in ordinary demand curve
$\frac{\partial x_1{}^U}{\partial P_1}$	Pure substitution effect--slope of compensated demand curve
$\frac{\partial x_1{}^M}{\partial P_2}$	Gross cross-price effect--shift in ordinary demand curve
$\frac{\partial x_1{}^U}{\partial P_2}$	Net cross-price effect--shift in compensated demand curve
$\frac{\partial x_1{}^M}{\partial P_1}+\frac{\partial x_1{}^M}{\partial M}x_1{}^M=\frac{\partial x_1{}^U}{\partial P_1}<0$	Slutsky equation expressed in linear effects
$\varepsilon_{11}^M+\varepsilon_{1M}S_i=\varepsilon_{11}^U<0$	Slutsky equation expressed in elasticities
$\frac{\partial x_1{}^M}{\partial P_2}+\frac{\partial x_1{}^M}{\partial M}x_2{}^M=\frac{\partial x_1^U}{\partial P_2}=\frac{\partial x_2^U}{\partial P_1}=\frac{\partial x_2{}^M}{\partial P_1}+\frac{\partial x_2{}^M}{\partial M}x_1{}^M$	Cross-price Slutsky equation
$\frac{\varepsilon_{12}}{S_2}+\varepsilon_{1M}=\frac{\varepsilon_{21}}{S_1}+\varepsilon_{2M}$	Cross-price Slutsky in elasticity terms
$\varepsilon_{11}+\varepsilon_{12}+\varepsilon_{1M}=0$	Homogeneity of ordinary demand function
$\frac{\partial x_1^U}{\partial P_1}P_1+\frac{\partial x_1^U}{\partial P_2}P_2=0$	Homogeneity of compensated demand function
$\varepsilon_{1M}S_1+\varepsilon_{2M}S_2=1$	Budget constraint requirement for income elasticities across ordinary demand curves
$(\varepsilon_{11}+1)\kappa_1+\varepsilon_{21}\kappa_2=0$	Budget constraint requirement for price elasticities across ordinary demand curves

BUDGET SHARES

The theory of consumer behavior is often expressed in terms of budget shares. From this perspective it is important to consider how income and price changes affect budget shares. Budget share can be expressed in terms of the demand function:

$$S_1 = \frac{p_1 x_1(p_1, p_2, M)}{M}$$

First, let's diffferentiate with respect to income:

$$\frac{\partial S_1}{\partial M} = \frac{p_1}{M}\frac{\partial x_1}{\partial M} - \frac{p_1 x_1}{M^2}$$

Now consider what it means if the budget share remains constant when income changes. If $\frac{\partial S_1}{\partial M} = 0$ then $\frac{p_1}{M}\frac{\partial x_1}{\partial M} - \frac{p_1 x_1}{M^2} = 0 = \frac{M}{x_1}\frac{\partial x_1}{\partial M} - 1$, or in other words, income elasticity is equal to one:

$$\frac{M}{x_1}\frac{\partial x_1}{\partial M} = 1 .$$

Similarly for price elasticity:

$$\frac{\partial S_1}{\partial p_1} = \frac{p_1}{M}\frac{\partial x_1}{\partial p_1} + \frac{x_1}{M} .$$

If $\frac{\partial S_1}{\partial p_1} = 0$ then $\frac{p_1}{M}\frac{\partial x_1}{\partial p_1} + \frac{x_1}{M} = 0$, or own price elasticity equals minus one:

$$\frac{p_1}{x_1}\frac{\partial x_1}{\partial p_1} = -1$$

If income and own price elasticities equal 1 and –1, by the homogeneity condition, the cross price elasticity must be zero.

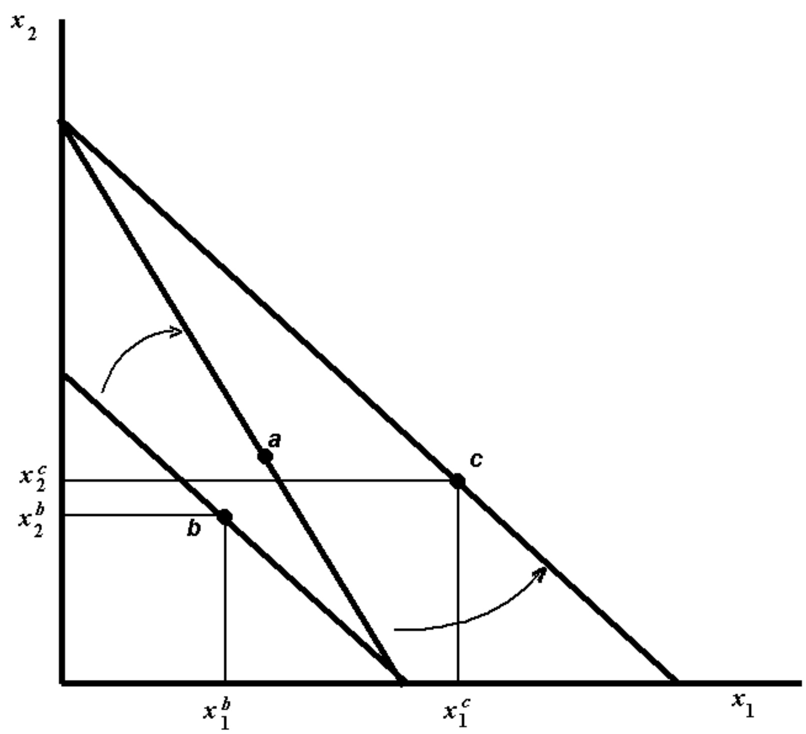
A Graphical Presentation of Result 2.2[8]

If the cross-price elasticity of *i* for *j* is negative while *j* for *i* is positive, then good *i* (the complement to *j*) must have a higher income elasticity.

Consider two goods as shown below. Let the price of good 2 go down which causes the individual to change the consumption choice from point *b* to point *a*. In so doing, good 1 is revealed to be a complement to good 2 (i.e., $\partial x_1/\partial p_2<0$).

Next consider a price decrease in good one. This causes the individual to change the consumption choice from point *a* to point *c*. In so doing, good 2 is revealed to be a substitute for good 1 (i.e., $\partial x_2/\partial p_1>0$).

Finally, notice that the first and third budget constraints are parallel. Hence, the movement from point *b* to point *c* measures an income effect. Also notice that based on the alignment of the points, the complement (good 1) has an income effect that is necessarily larger than the substitute (good 2).



[8] Thanks go to Haizhen Li for this presentation.

Setup	**Slutsky**	**Symmetry**	**Homogeneity**	**Budget Constraint**
$\max_{\{x_i, i=1,n\}} U(x_1,\ldots,x_n)$ $s.t.\ \ M^0 = \sum_{i=1}^{n} P_i x_i$	Law of Demand: Substitution effect is neg. Downward sloping compensated demand.	Cross price effects from compensated demand curves are equal.	Demand Curves are homogeneous of degree zero in money and prices.	Across Demand Conditions
FOC & SSOC => $x_i^M = x_i^M(P_1,\ldots,P_n,M^0)$ Ordinary Demand Curves	$\frac{\partial x_i^M}{\partial P_i} + \frac{\partial x_i^M}{\partial M} x_i^M = \frac{\partial x_i^U}{\partial P_i} < 0$ $\varepsilon_{ii}^M + \varepsilon_{iM} S_i = \varepsilon_{ii}^U < 0$	Cross Slutsky $\frac{\partial x_i^U}{\partial P_j} = \frac{\partial x_j^U}{\partial P_i}$ $\frac{\partial x_i^M}{\partial P_j} + \frac{\partial x_i^M}{\partial M} x_j^M = \frac{\partial x_j^M}{\partial P_i} + \frac{\partial x_j^M}{\partial M} x_i^M$	Euler's Theorem $\sum_{i=1,\ldots,i,\ldots n} \left[\frac{\partial x_i^M}{\partial P_i} P_i \right] + \frac{\partial x_i^M}{\partial M} M = 0$ $\sum_{i=1,i,n} \varepsilon_{ij} + \varepsilon_{iM} = 0$	Differentiate the budget constraint at the optimal values with respect to a price or income.
Duality: For every maximization problem, there is a mirror-image minimization problem.	Can't consume too much or spend too much: $-\varepsilon_{ii}/\varepsilon_{iM}$	Ordinary cross price effects are not necessarily symmetric If gross cross price effects are equal income *elasticities* must be equal.	Elasticities in ordinary demand curve must sum to zero. Empirical test Omitted variable bias	$\sum_{i=1}^{n} \varepsilon_{iM} S_i = 1$ $\varepsilon_{01} = \frac{S_1}{1-S_1}(\lvert \varepsilon_{11} \rvert - 1)$
$x_i^U = x_i^U(P_1,\ldots,P_n,U^0)$ Compensated Demand	For: $x_i = a + bP_i + cM + dP_j$	$\frac{\varepsilon_{ij}}{S_j} + \varepsilon_{iM} = \frac{\varepsilon_{ji}}{S_i} + \varepsilon_{jM}$		For elastic goods, substitutes dominate complements; for inelastic goods, complements dominate substitutes.
If constraints are matched: Duality Theorem $x_i^M(.) = x_i^U(.)$	$x_i < -b/c$ must be true for normal price and income effects.	Implications for relative income elasticities Graph		$\frac{\partial S_1}{\partial M} = 0 \Rightarrow \frac{M}{x_1} \frac{\partial x_1}{\partial M} = 1$ $\frac{\partial S_1}{\partial p_1} = 0 \Rightarrow \frac{p_1}{x_1} \frac{\partial x_1}{\partial p_1} = -1$