

The Paradigm of Consumer Behavior

How Dog Haters are like Dog Lovers

Dog Haters

- Just keep ‘em alive
- Do it Efficiently -- Known Nutrition Function
- Expenditure Minimizing Behavior

$$\min_{\{x_i\}} E = \sum_{i=1}^n p_i x_i + \lambda [N^0 - N(x_1, \dots, x_n)]$$

Dog Hater Optimal Choices

- FOC & SSOC:

$$\frac{\partial E}{\partial x_i} = p_i - \lambda N_i(.) = 0, i = 1, n$$

$$\frac{\partial E}{\partial \lambda} = N^0 - N(.) = 0$$

$$H = \begin{vmatrix} -\lambda & N_{ij} & \vdots & -N_j \\ \dots & & & \dots \\ -N_i & \vdots & & 0 \end{vmatrix}, i \& j = 1, n$$

Dog Hater Demand Curves

$$\begin{bmatrix}
 -\lambda N_{11} & \cdots & -\lambda N_{1i} & \cdots & -\lambda N_{1n} & -N_1 \\
 \vdots & \ddots & \vdots & & \vdots & \vdots \\
 -\lambda N_{i1} & \cdots & -\lambda N_{ii} & \cdots & -\lambda N_{in} & -N_i \\
 \vdots & & \vdots & \ddots & \vdots & \vdots \\
 -\lambda N_{n1} & \cdots & -\lambda N_{ni} & \cdots & -\lambda N_{nn} & -N_n \\
 -N_1 & \cdots & -N_i & \cdots & -N_n & 0
 \end{bmatrix}
 \begin{bmatrix}
 \partial x_1^* / \partial p_i \\
 \vdots \\
 \partial x_i^* / \partial p_i \\
 \vdots \\
 \partial x_n^* / \partial p_i \\
 \partial \lambda^* / \partial p_i
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 \vdots \\
 -1 \\
 \vdots \\
 0 \\
 0
 \end{bmatrix}$$

$$\frac{\partial x_i^*}{\partial p_i} = (-1) \frac{H_{ii}}{H}$$

Dog Lovers

- Fat 'n Happy
- Maximize Nutrition
- Subject to an Expenditure Constraint

$$\max_{\{x_i\}} N = N(x_1, \dots, x_n) + \mu \left[E^0 - \sum_{i=1}^n p_i x_i \right]$$

Dog Lover Optimal Choices

- FOC & SSOC:

$$\frac{\partial N}{\partial x_i} = N_i(\cdot) - \mu p_i = 0; i = 1, n$$

$$\frac{\partial N}{\partial \mu} = E^0 - \sum_{i=1}^n p_i x_i = 0$$

$$H^E = \begin{vmatrix} N_{ij} & \vdots & -p_i \\ \dots & & \dots \\ -p_j & \vdots & 0 \end{vmatrix}$$

Dog Lover Demand Curves

$$\begin{bmatrix} N_{ij} & \vdots & -p_i \\ \dots & & \dots \\ -p_j & \vdots & 0 \end{bmatrix} \begin{bmatrix} \partial x_k^E / \partial p_i \\ \dots \\ \partial x_i^E / \partial p_i \\ \dots \\ \partial \mu^E / \partial p_i \end{bmatrix} = \begin{bmatrix} 0 \\ \dots \\ \mu \\ \dots \\ x_i \end{bmatrix}$$

$$\frac{\partial x_i^E}{\partial p_i} = \mu \frac{H_{ii}^E}{H^E} + x_i \frac{H_{n+1,i}^E}{H^E}$$

Dog Haters v. Dog Lovers

- Haters:

$$x_i^N = x_i^N(p_1, \dots, p_n, N^0)$$

- Lovers:

$$x_i^E = x_i^E(p_1, \dots, p_n, E^0)$$

Dog Hater Hires a Dog Lover

- Hater's Budget becomes Lover's Constraint

$$E^* = \sum_{i=1}^n p_i x_i^N$$

- Commodity Choices are Identical (Duality)

$$x_i^E(p_1, \dots, p_n, E^*(p_1, \dots, p_n, N^0)) \equiv x_i^N(p_1, \dots, p_n, N^0)$$

- Demand Slopes Embody Symmetry

$$\frac{\partial x_i^E}{\partial p_i} + \frac{\partial x_i^E}{\partial E} \frac{\partial E^*(N^0)}{\partial p_i} = \frac{\partial x_i^N}{\partial p_i}$$

Dog Lovers act like Dog Haters

- Envelope Theorem: $E^* = \sum_{i=1}^n p_i x_i^N$
 $\frac{\partial E^*}{\partial p_i} = x_i^N = x_i^E$

- Behavioral Equivalence =>
LAW OF DEMAND:

$$\frac{\partial x_i^E}{\partial p_i} + \frac{\partial x_i^E}{\partial E} x_i^E = \frac{\partial x_i^N}{\partial p_i} < 0$$