Question:

Consider an individual with a utility function for consumption and leisure that is described as:

$$U = C \cdot l$$

where C is consumption and l is leisure. Assume that the individual has $25,000 in non-labor income and receives a wage rate of $50 per hour. Let the total time available to this person be 4992 hours per year to allocate between work and leisure. (4992 is 16 hours per day, 6 days per week, 52 weeks per year.) However, the individual is not perfectly free to choose a work schedule. The individual must work a minimum of 2250 hours per year to keep the job which pays $50.

a. How much does the person work and consume now? Is the constraint binding?
b. How much would the person work and consume at a wage of $100?
c. Calculate how much, if anything, this person would pay (on an annual basis) to be promoted to a job that pays $100 per hour.

Answer:

a) Set the problem up as normal utility maximization:

$$\max_{\{C,l\}} U = C \cdot l + \lambda(C - E - w[T - l]) \text{ .}$$

The FOC are the budget constraint and C = wl. Substitute for C in the budget constraint and solve for l^*. Then solve for C^*.

Unrestricted the person would consume 2746 hours of leisure at $50 wage. The constraint is binding because at 2250 hours of work, there are only 2742 hours for leisure. Constrained consumption is $137,500.

b) Same set up with the same FOC. Solve the same way. At $100 wage, the person demands 2621 hours of leisure. The constraint is not binding.

c) The problem of how much the individual will pay for the higher wage is solved by minimizing non-labor income at the utility level given in part (a).

$$\min_{\{C,l\}} E = C - w(T - l) + \phi(\overline{U} - C \cdot l)$$

Note that this is the constrained consumption level C` given by 2250 times $50 plus $25,000, and the constrained leisure level, $l$`, of 2742. That is,

$$\overline{U} = C`\cdot l` = \$137{,}500 \cdot 2472$$

In this problem, C^*=$194 and 200; l^*=1942. From these we can calculate E^*, which is negative $110,857. Labor income is $305,000.

This E^* is the non-labor income that will keep the individual on the utility indifference curve identified in part (a). Because the individual receives $25,000 of non-labor income each period, the person would be willing to pay this $25,000 plus E^* of $110,857 for a total of $135,857 each period for the wage of $100. A payment of a smaller amount will allow the individual to enjoy more utility than was found in part (a).

This is shown in the figure below. Point *a* represents the behavior found in part (a) above. Point *b* represents the choices in part (c). The distance *x z* is $25,000, the non-labor income received by the individual. The distance *z y* is the non-labor income that will keep the individual on the same indifference curve. Hence, the individual would be willing to pay up to the amount depicted by the distance *x y* for the higher wage.